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Remarks on Perturbative Categorical Quantum Gravity

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Motivations

Feynman diagrams are typical tools of QFT formulated on flat Minkowski spacetime M^4. They describe interactions of all quantum fields in the Standard Model, i.e. except *gravity*, in the perturbative regime.
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However, Feynman (1950-64) tried to build the theory of quantum gravity based on the Feynman diagrams on flat M^4.
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Linearized perturbations of metric undergoe quantization as other fields.
Whole approach is on *flat* M^4, so it is called *covariant* QG (i.e. Lorentz invariant) and *perturbative* – there are Feynman diagrams involved.

Main features of the *perturbative covariant QG*

i. Feynman diagrams are prime ingredients (not any classical gravity);
ii. It is a perturbative theory;
iii. The background is entirely flat Minkowski spacetime (hence no classical gravity = curvature);
iv. Classical GR is restored from the tree level of the diagrams [Feynman, 1962].

General setup

- On Minkowski ST, M^4, we have quantum fields representing elelctrons, photons, protons, gluons etc. They undergo quantization.
- Gravitational field is also defined on M^4 $(g_{\mu\nu})$ but it does not curve the spacetime. The linearization of $g_{\mu\nu}$, i.e. $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$, is quantized as other fields and gives rise to gravitons $G_{\mu\nu}$.
- Gravitational interactions are described by Feynman diagrams of QFT on flat Minkowski M^4 via interchanging the gravitons.
- The Feynman diagrams are built according to the rules:
  1. vertices
  2. propagators

Problems of pQG

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Currently we do not have any good (finite, renormalizable, perturbative or non-perturbative) theory of QG in dimension 4. However, pure QG is renormalizable on-shell in 1-loop, and QG with sources is non-renormalized in 1-loop [Veltman, t'Hooft,1970's].

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Instead, let us take a reasonable categorical extension of 4-D spacetime and try to carry out the Feynman project on it.

A Categorical perturbative QG

We work in a model of Synthetic Differential Geometry (SDG), i.e. some smooth topos $\mathcal{T}$ in which the category of smooth manifolds and maps, $\mathbb{M}$, is represented fully and faithfully

$$s : \mathbb{M} \hookrightarrow \mathcal{T}.$$

$s(\mathbb{R}) = R$ contains various objects of infinitesimal 'numbers' ($\neq 0$)

$$D_k \to_{\mathcal{T}} R, d \in D_k | d^{k+1} = 0.$$

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A categorical spacetime is a spacetime M^4 which 'locally' (for sufficiently fine local cover $\{U_\alpha \simeq \mathbb{R}^4\}_{\alpha \in I}$) allows the internal to $\mathcal{T}$ reasoning, i.e.

$$\forall U_\alpha \exists U'_\alpha \subset U_\alpha \, (s(U'_\alpha) \simeq_{\mathcal{T}} R^4).$$

The point is that we start with M^4 in SET at macro-scale, and continue in $\mathcal{T}$ at micro-scale. PHYSICS: in sufficiently high energies (small distances) the reasoning becomes intuitionistic and fields and spacetime live in $\mathcal{T}$.

Monads

In $\mathcal{T}$ there are monads $\mathcal{M}(x), x \in s(M^4)$ in particular around $s(x)$ in $\mathcal{T}$ such that $x \in M^4$ in SET, i.e.

$$\forall x \in M^4 (\mathcal{M}_k(x) = \{y \in s(M^4) | y - x \in D_k^4\})$$

where we refer to the local (micro)linear structure for $-$.
$\mathcal{M}_k(x)$ is parametrized by D_k for some $k \in \mathbb{N}$. Note that

$$d^{k+1} = 0, d \in D_k.$$

Certainly in SET there are no $d \in \mathbb{R}$ that $d^{k+1} = 0, d \neq 0$.
So, we need intuitionism and topoi.

Gravity on monads

MAIN QUESTION: given monads $\mathcal{M}(x) \hookrightarrow s(M^4)$ in $\mathcal{T}$ does gravitational field $h_{\mu\nu}(x), x \in M^4$ in SET know about them?
Can $h_{\mu\nu}$ propagate on monads?

The ANSWER: the only field propagating on monads is $h_{\mu\nu}$. It is the only field 'sensitive' on the intuitionistic structure of the spacetime at microscale.

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– *Quantum* gravitational interactions take place on monads in $\mathcal{T}$ so that the vertices with virtual gravitons live on monads.
– *Classical* gravity is described entirely on M^4 in SET.

/This discrepancy $\Longrightarrow$ problems with quantizing gravity?/

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SDG: D_k is described by the spectrum, $Spec_R(W)$, of a Weil algebra W and W can be represented as $W = \mathbb{R} \oplus \mathbb{R}[\epsilon] \oplus \mathbb{R}[\epsilon^2] \oplus ...$ (k-many ϵ terms) and the real spectrum is $\mathbb{R}[\epsilon] \oplus \mathbb{R}[\epsilon^2] \oplus ....$ This is how D_k looks like in SET.

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ϵ is the so called dual number, $\epsilon^{k+1} = 0$.

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If (quantum) gravity interacts at monads we can describe it in SET via W picture. In infinitesimal terms on D_k the gravitational coupling constant $f = \sqrt{8\pi G}$ is represented by $\sqrt{8\pi G} \cdot \epsilon$. Physics distinguishes D_2, so ϵ is from D_2, thus

$$(\sqrt{G} \cdot \epsilon)^3 = 0$$

and

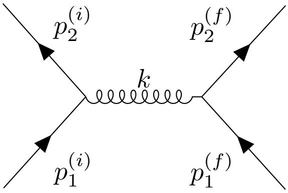
$$f_{D_2} = \sqrt{8\pi G} \cdot \epsilon$$

Since only gravity explores monads this is the only coupling with ϵ unit.

We will see that Feynman covariant QG on the categorical spacetime has very desirable features.

perturbative QG on monads and classical Newton law

The covariant term for the interchanging of a single graviton (in the lowest tree-level) on a monad is calculated from the diagram



$$-if^2 T^1_{\mu\nu} \frac{\mathcal{P}^{\mu\nu,\alpha\beta}}{k^2 + i} T^2_{\alpha\beta} \delta(P_{fi}) \cdot \epsilon^2.$$

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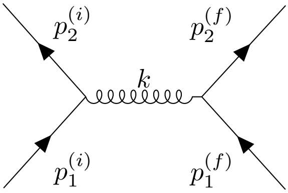

$$-if^2T^1_{\mu\nu}\frac{\mathcal{P}^{\mu\nu,\alpha\beta}}{k^2+i}T^2_{\alpha\beta}\delta(P_{fi})\cdot\epsilon^2.$$

The Fourier transform of it (T) (in the Mandelstam coordinates, non-relativistic) reads

$$-\frac{1}{m_1m_2}\int e^{-i\mathbf{k}\cdot\mathbf{r}}(T)d^3k\cdot\epsilon^2=\big(-\frac{f^2m_1m_2}{8\pi r}+\frac{1}{2}f^2\delta^3(\mathbf{r})\big)\cdot\epsilon^2.$$

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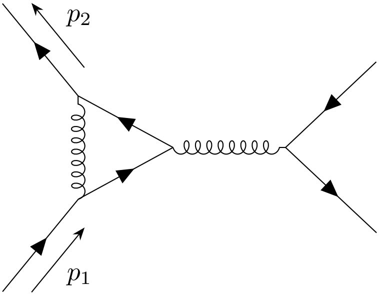

where $\sim f_{D_2}^4 = 0$.

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Where is classical curvature–gravity on flat Minkowski M^4? Do we rule out it completely?

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ANSWER: Yes. However the smoothness on manifolds like $M^3 \times \mathbb{R}$ has to be non-standard (exotic) in SET, if $M^3 \times \mathbb{R}$ is to be categorical spacetime (i.e. allows for the change SET $\to \mathcal{T}$ at microscale) [JK,MH,2017].

This is the consequence of incompatibility of the real line $\mathbb{R}$ in SET with the intuitionistic real line $R_{\mathcal{T}}$ in $\mathcal{T}$.

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The meaning of this 'exotic' curvature is physical and is under intense study currently [T.Asselmeyer-Maluga,JK,2017].

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(–) Exotic R^4 can not be Riemann flat. Its curvature is physical:
the curvature of $R^4 \sim$ the measured value of the CC density energy of the expanding universe.
(To explain this value is one of the biggest, still open, mysteries of physics).

a bit of formal play

Superspace Grassmann's coordinates Θ fulfill

$$\{\Theta, \Theta\} = [\Theta, \Theta] = 0 \Longrightarrow \Theta^2 = 0. \tag{1}$$

Maybe we should work in yet another coordinates, $d^3 = 0$, in order to quantize gravity? (ϵ-valued physical units)

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One way is easy, if $\Theta = d^2$ then (1) holds, and formally

$$d = \sqrt{\Theta}...$$

Thank You!